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Learning and Decision Model Selection for a Class of Complex Adaptive Systems

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Abstract

Computer modeling is gaining popularity in the study of systems whose underlying processes are difficult to observe and measure directly, or their controlled experimentation is not an option. Since real-world phenomena, for instance psychological or ecological, are often hugely complicated, and the models trying to capture their essence relatively complex, validation of the models and selection among the candidates is a challenge. Furthermore, not all computer models are used merely for explanatory purposes or to test theories, but some are used to support decision making. Therefore, it is critical which model the decision makers put their confidence on. In this article I discuss a pragmatic method for selecting between classes of models that are designed to increase understanding in the most significant single factor behind the global climate change, namely human land-use. My focus is on agent-based land-use and land-cover change models, and particularly models of learning and decision making. The proposed method fills the void left by traditional statistical model selection methods that are not applicable due to the nature of the model class of interest.

Keywords: Agent-based modeling; model selection; minimum description length principle; decision making.

Introduction

These days Earth's land-cover is going through changes at faster pace than ever, and most of these changes are human initiated. Pervasive land-use and consequent land-cover changes, occurring in different time scales and spatial extent, have had and continually have adverse impact on local, regional and global level by destroying natural ecosystems and causing irreversible changes in global climate. In order to understand the impact land-use change has on ecological systems, not only its consequences but also the underlying mechanisms and forces driving land-use decisions need to be explained.

Empirical measurements are not sufficient to understand the combination of the factors behind the change (Parker, Manson, Janssen, Hoffman, & Deadman, 2003). On the other hand, experimental manipulation of landscapes is often impractical if not impossible (Baker, 1989). Combined with other methods, for instance household surveys and analysis of census data, computer models offer a relatively effortless method for testing alternative theories and formulating new hypotheses, analyzing implications of environmental policies, predicting changes and exploring interactions between, for instance, social, psychological, economical, bioecological, and even political and historical factors behind land-use.

A number of different techniques have been used in modeling the *land-use and land-cover change (LUCC)* (Parker et al., 2003), for instance equation-based models, logistic regression models based on suitability maps (Schneider & Pontius, 2001), system dynamic models, statistical methods, symbolic or rule-based systems combined with qualitative expert knowledge, and evolutionary models, such as genetic algorithms. Perhaps perhaps the most common methods are cellular automata (CA) and Markov chain (MC) models, or combinations of them (Brown, Riolo, Robinson, North, & Rand, 2005; Jenerette & Wu, 2001; Parker et al., 2003).

Most of the early modeling efforts have concentrated in biophysical processes rather than human actions (Itami & Gimblett, 2001), even if the majority of the land-use change is initiated by humans. On the other hand, mathematical and statistical methods ignore the spatial aspect of LUCC (Manson, 2000). Therefore, in this article I consider a type of models that still is an emerging approach, namely a combination of a cellular model representing the biophysical landscape, and an *agent-based* component representing the decision makers, either individuals, households or institutions. Land-use is then what links the agent to the landscape (Parker et al., 2003; Evans, Sun, & Kelley, 2006).

Since computer models are often used to inform decision makers in the process of designing environmental programs and policies, and the direct or indirect consequences of these decisions may be consequential, models' plausibility and adequacy to the task needs to be rigorously assessed, i.e., it is pivotal to have a right model to the task. Models may generate seemingly plausible outcomes even if the generating mechanism is quite arbitrary. On the other hand, proper tweaking of parameter values may make them produce any results the decision maker would like to see. The lack of adequate tools often makes it difficult to compare and choose between alternative models on a fair basis without relying on their face value, i.e., how well the model behavior confirms to the decision maker's ideals. Therefore, it is important that the choice of the model that decision makers put their confidence on is based on sound principles. In other words, the evaluation, validation and selection methods are as crucial as the models themselves.

Several different model selection methods, such as *Akaike's Information Criterion (AIC)* (Akaike, 1973), *Bayesian Information Criterion (BIC)* (Schwarz, 1978), and the *Minimum Description Length (MDL)* principle (Grünwald, 1998), particularly its enhanced version *Normalized Maximum Likelihood (NML)* distribution (Rissanen, 1999), apply to probabilistic model classes. However, LUCC models do not lend themselves easily to probabilistic interpretation but can be best characterized as *complex adaptive systems (CAS)*. Moreover, land-use change data is not always readily available in quantities warranting use of cross-validation or bootstrap methods (Lendasse, Wertz, & Verleysen, 2003).

In this article I study a model selection method based on a practical interpretation of the MDL principle. In the next chapter I review the agent-based framework for LUCC modeling. Discussion on the model selection criterion follows. The criterion was originally introduced and extensively evaluated with a set of artificial data in Laine (2006). Here its properties are addressed in the context of real-world data.

Agent-based Models of Land-use and Land-cover Change

Two fundamental ideas behind *agent-based models (ABMs)* are: first, the decision making is distributed among autonomous actors, which either operate individually or may communicate and cooperate, and secondly, the heterogeneity of actors is captured by characteristics that may be unique or shared by agents. The focus is on the macro-level patterns in collective behavior emerging from agents' individual characteristics and micro-level phenomena, such as local behavior and interaction between agents.

ABMs come in multiple disguises but here I am particularly interested in models in which agents inhabit a simulated environment, so that they are 'physically' tied to a specific location and have a fixed neighborhood. The models of land-use and land-cover change fall into this category of models.

The agent-based approach has been used to study various land cover change related processes in several areas of the world: for instance agricultural land-use decision making by colonist households in Brazilian Amazon (Deadman, Robinson, Moran, & Brondizio, 2004), migration and deforestation in Philippines (Huigen, 2004), agricultural household land-use decision making in the US Midwest (Evans & Kelley, 2004; Laine & Busemeyer, 2004), reforestation in the Yucatan peninsula of Mexico (Manson, 2000), and ex-urban development in Maryland, US (Irwin & Bockstael, 2002).

Land-use Framework

The conceptual assumptions behind the land-use framework were adapted from Cioffi-Revilla & Gotts (2003). The most important ones are listed below:

1. The *landscape* is an abstract rectangular area divided into *cells* of equal size, which serve as the decision-making units.

2. Each cell has various biophysical properties that remain constant over time.

3. The main actors in the model are autonomous *agents*. They have a potentially infinite existence, although they can perish. All agents are of the same type (e.g., households), but their individual characteristics may vary.

4. Agents control a region, called *parcel*, which is a set of adjacent cells on the two-dimensional landscape. Agents have exclusive access to this region, and there is no property exchange between the agents.

5. Agents make resource allocation decisions on their parcel in order to satisfy their goals. Agents have a limited set of available actions, i.e., options to which to allocate their resources. Agent actions change the use of the cells on their parcel.

6. All agents have the same learning and decision strategy.

7. The global environment consists of external conditions that are common to all parcels. These conditions may change over time.

The architecture of the system is depicted in Figure 1.

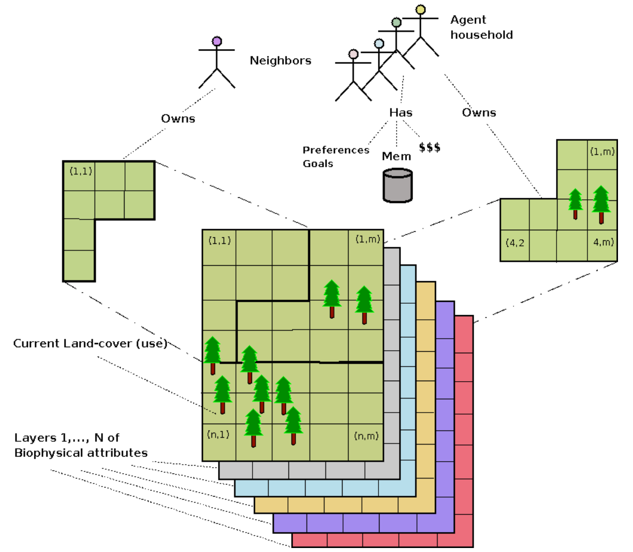

Figure 1: Main components of the land-use framework.

Decision Models

At each decision round agents observe the state of their land, and make a decision about its use in the next round. They make the decision for each cell separately; they either decide to keep the old use or select another use from the given alternatives. After making the decision for each cell, they observe the payoff earned from different uses. This payoff is then used as a basis for the next decision.

In this study I am primarily interested in agents' learning and decision processes. Thus, the alternative model classes in selection consist of different decision and learning strategies. In addition to a random and a null model (which never makes any changes), other model classes chosen for the study constitute a set of relatively straightforward reinforcement-based strategies, familiar from psychology and economics literature. These are a model that makes locally greedy changes, Q-learner (Watkins & Dayan, 1992), and two versions of the *experience-weighted attraction (EWA)* model (Camerer & Ho, 1999): one that only considers its individual payoff (iEWA), and one that also takes its neighbors' payoff into account (sEWA).

Model Selection Framework

Characteristic to the class of LUCC models, as opposed to more traditional cognitive models, is that they are often validated against land-use data instead of comparing the model's behavior to experimental human data. The modeling task then is to find out what kind of decision processes may have generated the observed land-use change patterns. This indirect derivation of agent behavior from the landscape poses another range of challenges to the validation process. Yet another validation technique emerging in LUCC modeling is field experiments, in which the researcher takes her laboratory to the stakeholders and makes them play a role game that mimics the real-world decision making context (Olivier Barreteau & Attonaty, 2001; Carpenter, Harrison, & List, 2005).

Challenges to the Model Selection Criterion

So, which method should be used to select between agent-based LUCC models? There is no straightforward answer, but several inherent characteristics of the modeling domain needs to be taken into consideration. These challenges, more thoroughly discussed in Laine (2006), are reviewed next.

First, with the exception of some simple cases[1], it is dangerous to assume that some 'true' model exists, and design a system so that it tries to approximate this 'truth'. After all, model parameters and functions are not inherent properties of the system we want to model but theoretical constructs we use to describe the system. We impose the properties to the system. Again, there is no way to verify that a 'true model' exists, and consequently the task of estimating something that does not exist becomes quite impossible.

Secondly, existing model selection methods most commonly penalize for model complexity[2], i.e., its propensity to overfit, by taking the number of free parameters into account. A typical LUCC model is a collection of multiple autonomous components and processes that interact at multiple spatial levels and temporal scales. Thus, free parameters are not equally easy to identify in this class of models as they are in probabilistic or polynomial model classes.

Thirdly, the data available for the validation of CAS are not plenty and always not random samples. Sometimes it is even hard to make a distinction between the data and the model.

These considerations make it particularly clear that most of existing model selection methods, for instance penalized maximum likelihood methods, such as AIC or BIC, are inapplicable. Nevertheless, the MDL principle, and especially its refined formulation, the NML distribution, have some nice theoretical properties, but for many practically interesting model classes they cannot be calculated (Rissanen, 1999). Finally, in many cases the scarcity of data does not allow for adequate generalization tests.

Normalized Minimum Error Principle

Here I propose a selection criterion that overcomes some of these challenges. It makes the following assumptions:

- No 'true' model exists.
- Measure of flexibility is based on the model's performance with respect to data, not some predetermined structural property.
- A model itself does not determine its fit to data, but an error function is required.

While the last two points address the trade-off between goodness-of-fit and the model class flexibility, the first one takes a more ideological standpoint on what is tried to achieve with the model selection criterion, namely that the goal is to find the best model to explain the data rather than a model that approximates some 'true' state of the world. We need to estimate the model's fit in order to quantify how well it captures the essential properties of the data.

The fit is not enough, since too flexible model is prone to overfit. *Two-part code*, also called a *crude version* of the MDL principle trades off flexibility to superior fit by choosing the model H in class $\mathcal{M}_i$ that minimizes the sum $L(D|H,\mathcal{M}_i)+L(H|\mathcal{M}_i)$, where $L(\cdot)$ is the description length in bits. The underlying idea is that regularities in the data can be used to compress it, and the best model to explain the data is one that compresses the data most efficiently. In other words, the model using the least number of bits in describing the data most likely captures its underlying regularities. These regularities can then be used to gain insight on the structures and processes that generated the data.

The two-part code formulation still uses the maximum likelihood parameters to account for the model class flexibility (the second term in equation). We are not interested in the best-fitting model, but a well-fitting model in a class that is not overly flexible. In other words, we want to find a model that can reveal interesting patterns in the data, not a model that captures mere noise. This is where the error function comes into play. Next, I will present a method how to treat the trade-off between fit and flexibility adequately using errors.

If we want to explain an observed data sample x^n from the set of all data samples X^n with the help of the model class $\mathcal{M}_i$, ideally we want $\mathcal{M}_i$ to

1. contain a model H that makes a small error on x^n, and
2. contain models H' that do not make small errors on most y^n belonging to X^n.

[1] Simple cases such as the model of the average height of six graders, or presidential candidate's approval rate.

[2] Following the terminology adopted in Laine (2006), I substitute the term 'flexibility' for 'complexity' for two reasons; first, the latter is heavily burdened, meaning different things for different people, and secondly, the LUCC model class and the modeled domain are inherently complex systems, so it would be misleading to imply that complexity is necessarily problematic.

This can be achieved by minimizing the following ratio, called *Normalized Minimum Error (NME)* (Laine, 2006):

$$NME(x^n, \mathcal{M}_i) = \frac{ER(x^n|\hat{\theta}(x^n, \mathcal{M}_i)}{\sum_{y^n \in X^n} ER(y^n|\hat{\theta}(y^n, \mathcal{M}_i))},$$

where $ER(\cdot)$ is the error model class $\mathcal{M}_i$ makes on x^n using the parameter values $\hat{\theta}(x^n)$ that minimize the error, and y^n are 'all possible data samples'. By normalizing each error this way we obtain a relative measure for fit and flexibility, which we can use as a model selection criterion.

The MDL principle is a general method of doing inductive inference, and the NME criterion is one way of implementing it. Yet another interpretation of the principle is the NML distribution, which selects a model class $\mathcal{M}_i$ whose *universal model* H, not necessarily in $\mathcal{M}_i$, minimizes the worst case regret. Regret of model H with respect to class $\mathcal{M}_i$ is the extra number of bits that are required to describe the data sample x^n using H instead of using x^n's maximum likelihood model in $\mathcal{M}_i$. H is called a universal model, since it tries to mimic all models in the class $\mathcal{M}_i$. It has been proved (Rissanen, 1999) that the NML criterion defines a unique model that minimizes the maximum regret.

The NME criterion uses errors as measure of fit, whereas the NML criterion uses probabilities. The term in the denominator is the most crucial aspect of both criteria, since it accounts for their ability to penalize for excess flexibility. The relationship between these two was demonstrated in Laine (2006).

Evaluation of the Criterion

The proposed criterion has been extensively tested with artificially generated data in Laine (2006). In this section I discuss some of its properties in the light of a representative case of real land-cover change data.

Review of Experiments with Artificial Data

Acquisition of multiple samples of accurate land-cover data with a good resolution is difficult or at least time consuming. Therefore, the preliminary experiments were conducted with data generated by an artificial system, i.e., the same model classes that were used as candidate models were also used as data generating classes. This is a common practice when comparing multiple model selection methods (Busemeyer & Wang, 2000; Pitt, Myung, & Zhang, 2002). The experiments were run in several conditions by varying the biophysical and agent characteristics, and the error function.

The main findings in the first set of experiments are:

1. The criterion tends to select the generating class if it is among the candidates.
2. The criterion predominantly selects model classes with fewer free parameters, and never chooses a class more flexible than the generating class.
3. For no data set it strongly prefers any single class, but the selected model depends on the error function.

Case-study

The data used in the second set of experiments comes from the state of Indiana in the Midwestern United States. The forest cover of the state of Indiana has undergone drastic changes during the last couple of hundred years; from almost 100% of the state being forest before the first settlers entered and cleared the land for agricultural production, down to 5-10% in the early 1900's, and then up to the current day's 20%, which mostly resides on the rolling hills of the South-central part of the state.

This study concentrates on deforestation and reforestation between 1940 and 1998 in two rural townships, Indian Creek and Van Buren, both about $10km \times 10km$ in size. The available data indicates that the forest cover has undergone a significant increase within the first 15 years of the study period and after that a modest but gradual increase. The overall increase of forest cover is around 20% in both townships. The change has not been unidirectional nor uniform; for instance, both deforestation and afforestation can be seen in the both townships, as pictured in Figure 2.

Data

Data used in these experiments consists of land-cover maps covering the study period, slope and soil data, and ownership data. In addition to these, economic data (prices and wages), and forest growth data were imported as exogenous forces. The land-cover is represented as a grid of cells of size $50m \times 50m$ that records the land-use for each cell. Ownership, slope, and soil data is recorded per cell in similar grids.

Experimental Conditions

The experiments were divided into a number of conditions by varying:

1. **Agent characteristics** Homogeneous vs. heterogeneous agents by household size, initial wealth and the number of neighbors.
2. **Fitting method** Landscape level vs. individual parcel level fit of parameters.
3. **Error function** (1) Mean absolute difference, (2) composition, (3) edge length, and (4) mean patch size. The first one measures the point by point difference between two landscapes, whereas the latter three calculate a squared difference between forest percentages, forest border lengths or mean forest patch sizes of two landscapes.

Results

The proposed model selection criterion cannot be analyzed in isolation of the error function it uses. The current study uses four different error functions three of which are so called summary statistics; they characterize a single aspect of the land-cover, whereas the fourth one, mean absolute difference, is a location by location measure. This metric uses more information of the landscapes than the other three that do not consider location.

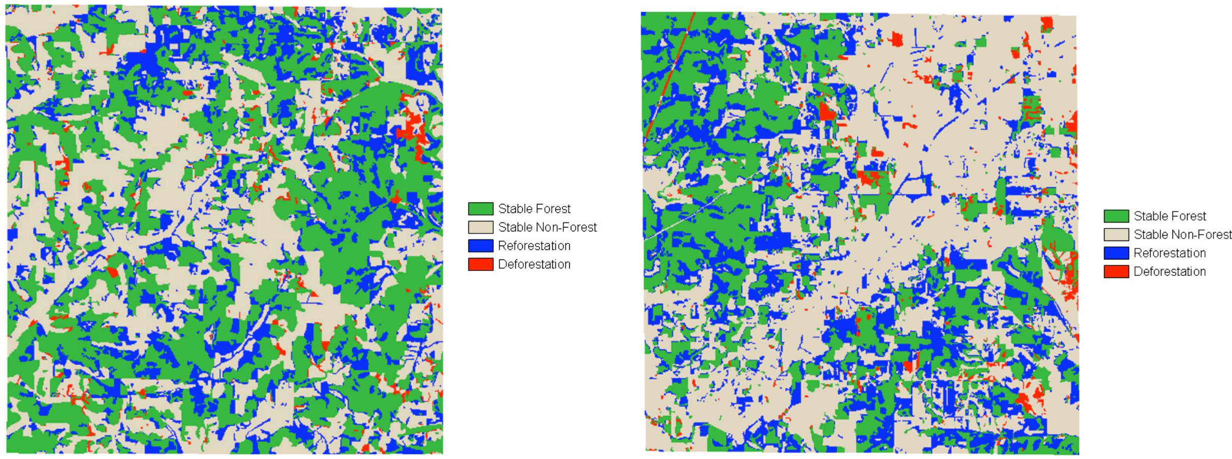

Figure 2: Deforestation, afforestation and stable forest cover in Indian Creek (left) and Van Buren (right) townships from 1940 to 1998.

	Indian Creek		Van Buren	
Error:	Selected	NME (μ)	Selected	NME (μ)
(1)	sEWA (c)	.25 (.413)	random	.499 (.588)
(2)	iEWA (c)	.05 (.415)	Q (c)	.12 (.585)
(3)	sEWA (i)	.35 (.463)	sEWA (c)	.05 (.537)
(4)	sEWA (c)	.103 (.406)	iEWA (c)	.49 (.594)

Table 1: Selected model classes and their NME scores for homogeneous agents with landscape level fit (mean scores in parenthesis, c=collectively fitted, i=individually fitted).

	Indian Creek		Van Buren	
Error:	Selected	NME (μ)	Selected	NME (μ)
(1)	sEWA (i)	.193 (.249)	iEWA (c)	.59 (.752)
(2)	greedy (c)	.04 (.180)	Q (c)	.39 (.820)
(3)	Q (i)	.154 (.205)	sEWA (c)	.67 (.795)
(4)	greedy (i)	.674 (.778)	Q (c)	.03 (.222)

Table 2: Selected model classes and their NME scores for heterogeneous agents with parcel level fit (mean scores in parenthesis, c=collectively fitted, i=individually fitted)).

Summary statistics are supposedly easier to fit, since there are several possible ways to get them right, for instance, several different land-cover configurations may have the same composition. Consequently, there are fewer ways of getting them wrong, too. However, there are very few ways, actually only one, of getting the location-by-location comparison correct, and a considerable number of ways of getting it wrong.

The selected models together with the respective NME scores and their means are presented in Tables 1 and 2 for homogeneous and heterogeneous agents, respectively, using different error functions. The number of decimal points is determined by how many decimals are needed to distinguish between the NME scores.

For homogeneous agents only one time out of eight is the individually fitted model class selected, whereas for heterogeneous agents three times out of eight. This is roughly what can be expected; when there is more variation in the agent population, there is potentially something to be gained by fitting the agents individually. In other words, the benefit attained in better fit outweighs the cost in extra flexibility.

In general, the selection criterion selects simpler models, i.e., collectively fitted classes, for homogeneous agents with both landscapes. However, with heterogeneous agents it predominantly selects individually fitted classes for Indian Creek, but collectively fitted for Van Buren. This indicates that either agent heterogeneity plays a bigger role in Indian Creek and some of the models classes are able to capture it, or the larger number of agents in Van Buren is hard to fit, and the selection criterion resorts to making a safe decision of selecting simpler model classes.

Finally, null and random model classes are seldom selected. This supports the fact that the real landscapes are dynamic, and undergo very characteristic changes which cannot be captured either by a chaotic or a stationary process.

Discussion and Future Work

The literature provides us with evidence that, somewhat counter-intuitively, location-by-location comparison is not that difficult after all. Pontius *et al.* (2004) argue that not a single model has been reported that is able to predict the location of land-cover changes better than a null model, a model that predicts no change. The proposed selection criterion is looking for a model class that is simple and contains a model that fits the data well. Since the changes over time in the real landscapes are usually small, a model that predicts few changes should perform well. Why does not the NME criterion select the null model?

In the current experiments 'all possible data' was replaced by 'all available data' for practical reasons. This decision has detrimental consequences. For instance, even if both Indiana landscapes exhibit some idiosyncrasies, nevertheless they can be assumed to be generated by 'the same process'; they are physically linked, subject to the same weather conditions and under the same county rules.

However, the NME criterion penalizes a model class, possibly the null model class, that fits well both of these data samples, as if it fitted 'all data' well, and never chooses the same model class for both landscapes. There is no outstanding solution to this dilemma yet. Thus, the very first theoretical and practical challenge is to circumscribe the actual meaning of 'all possible data' in order to fully understand the relation between theoretical underpinnings of the proposed criterion and the underlying practical issues inherent to the modeled domain.

Finally, although a common agreement in the field of LUCC modeling is that model validation is crucial, this study represents one of the first attempts to introduce model selection methodology to this complex spatial domain. The goal of model selection is to find a model that helps us gain insight into the processes underlying the — natural, psychological or economic — phenomenon of interest. Although the proposed criterion penalizes for excess complexity, simplicity is not the end in itself, but prevents us from becoming overconfident in more complex models when there is not enough data to support them. On the other hand, a considerable reflection should be involved when choosing the candidate models: too simplistic models to start with do not bring us any closer to understanding complex natural phenomena.

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